Mode choice models

Mode CHOICE

- Mode choice or mode split reflects travelers decision to select “how” to travel
- Reasons include level of service: time, cost, safety, convenience, security and individual characteristics: age, income, gender, trip type
- Three main factors: Mode characteristics, trip characteristics, trip maker characteristics

The utility functions

- UF measures the degree of satisfaction that travelers derive from their selection
- The magnitude depends on the characteristic of each choice and the characteristic of individual making that choice
- To construct a UF relevant variables as well as relevant form both are important
- Typically expressed as linear weighted sum of independent variables

$$U = a_o + a_1 X_1 + a_2 X_2 + \ \ldots + a_r X_r$$

Mode abstract and mode specific UF

- Mode abstract: value(weight) of independent variables remains constant
- Mode specific: mode specific value of time, cost etc. Example for three modes:

$$U_1 = 6.2 + 2.4X_1 + 3.5X_2$$

$$U_2 = 3.4 + 3.1X_1 + 2.9X_3$$

$$U_3 = 4.3 + 2.9X_1 + 3.2X_3$$

Mode choice models

$$U_m = \alpha + \beta t_m + \tau \quad c_m + \varepsilon.$$

We now assume that the utility is composed of two parts, a determinist part $U_m = \sum W_i S_{mi}$ and a random part ∈:

$$U_m = U_m + \in = \sum_{i=1}^{3} W_i S_{mi} + \in. \qquad (2.23)$$

The probability that the consumer chooses alternative 1, denoted by p_1, is

$$p_1 = prob(U_1 > U_2). \qquad (2.24)$$

Similarly,

$$p_2 = prob(U_2 > U_1). \qquad (2.25)$$

Under certain assumptions about the probability distribution of Ɛ – specifically that Ɛ is Weibull – distributed (Charles River Associates 1972, Ben-Akiva 1973, 1974, Mcfadden 1975) – the following results are obtained:

$$p_1 = \frac{e^{U1}}{e^{U1}+e^{U2}}, \qquad (2.26)$$

$$p_2 = \frac{e^{U2}}{e^{U1}+e^{U2}}, \qquad (2.27)$$

For the two-choice case it is instructive to define

G(S) = $u_1(S)$ - $u_2(S)$

So that (2.26) and (2.27) become

$$p_1 = \frac{1}{1+e^{U2-U1}} = \frac{1}{1+e^{-G(S)}},$$

$$p_2 = \frac{1}{1+e^{U1-U2}} = \frac{1}{1+e^{G(S)}},$$

Disaggregate Discrete Choice model (deterministic & stochastic)

- Two components of UF, known and ‘unknown’
- Unknown or ‘error’ in the model can have certain distribution
- If it is independent and identically distributed according to gumbel distribution(mean 0,SD σ) we get logit form:

$$P(k) = \frac{e^{U_k}}{\sum_x e^{U_x}}$$

Mode choice example

A calibration study resulted in the following utility equation:

$U = a - 0.025X_1 - 0.032\ X_2 - 0.015\ X_3 - 0.002X_4$,
Where X_1 = access and egress time in minutes, X_2 = waiting time in minute,
X_3= line haul time (min),
X_4= out of pocket cost in INR
The trip distribution forecast for target year shows Q = 5000 person trips per day. The target year service attributes for car and bus system are given as:
Assuming that the calibrated mode specific constants are 0.00 for car and -0.10 for bus, apply the logit model to estimate the target year market share of the two modes and the resulting fare box revenue for the bus system.

Attribute	X_1	X_2	X_3	X_4
car	5	0	20	100
Bus	10	15	40	50

Example cont……

Solution

The utility equation yields
U(A) = - 0.625 and U(B) = - 1.530

According to the logit equation),
p(A) = 0.71 and p(B) = 0.29

Therefore the market share of each mode is

$$Q_{IJ}(A) = (0.71)(5000) = 3550\, trips/day$$
$$Q_{IJ}(B) = (0.29)(5000) = 1450\, trips/day$$

The fare-box revenue estimate is
(1450 trips/day)(INR 20/trip) = INR 29000 per day

Incremental/pivot point model

$$P'(K) = \frac{e^{(Uk+\Delta Uk)}}{\sum_x e^{(Ux+\Delta Ux)}} = \frac{e^{U_k} e^{\Delta Uk}}{\sum_x e^{U_k} e^{\Delta Uk}}$$

Dividing the numerator and denominator by the denominator

$$P'(K) = \frac{\frac{\exp(U_k)}{\sum_x \exp(Ux)} X \exp(\Delta U_k)}{\sum_x \frac{\exp(Ux)}{\sum_x \exp(Ux)} X \exp(\Delta Ux)} = \frac{P(K) X e^{\Delta Uk}}{\sum_x P(x) X e^{\Delta Ux}}$$

The baseline probabilities do not need to be calculated by the MNL model; they may be obtained from surveys of existing conditions.

Independence of IRRELEVANT ALTERNATIVES property

$$\frac{P(A)}{P(B)} = \frac{\exp(U_A)}{\exp(U_B)} = \exp(U_A - U_B)$$

This ratio is not influenced by any change in the utility of the ("irrelevant") alternative.

Another way of stating this condition is that any change in the utility of the third alternative will affect the shares of A and B by the same proportion. This condition is known as the independence of irrelevant alternatives (IIA) property. To illustrate the IIA.

Counter intuitive results! Redbus/blue bus example

Nested Logit Model

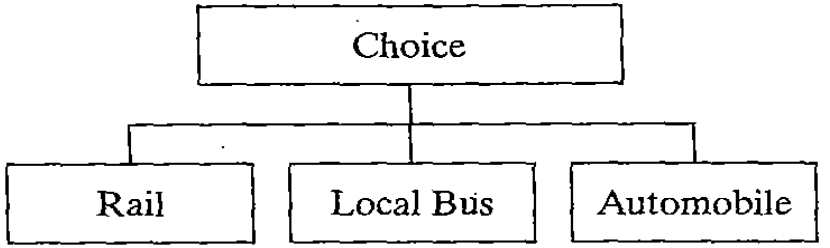
Hierarchical structure, clustering similar alternatives

$$P(A) = \frac{\exp(U_A)}{\exp(U_T) + \exp(U_A)}$$

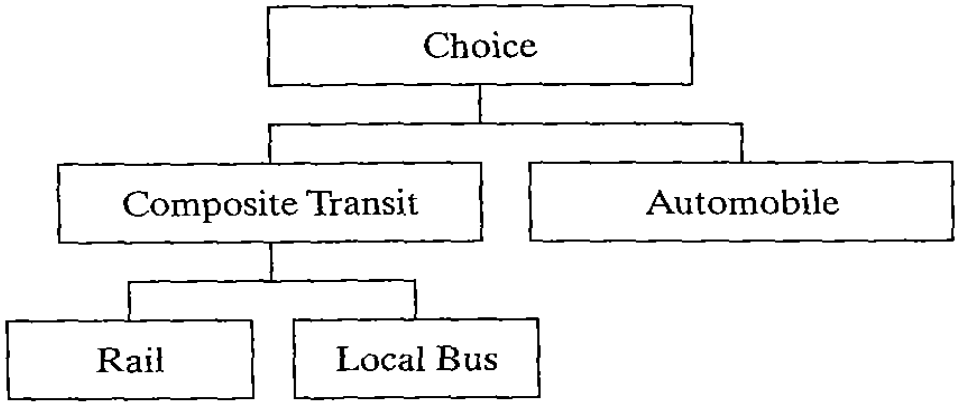
$$P(T) = \frac{\exp(U_T)}{\exp(U_T) + \exp(U_A)}$$

Where the composite transit utility

$$U_T = f(U_B, U_R).$$



(a) Multinomial logit structure for a three-mode choice.



(b) Nested Logit Structure

Nested Logit Model cont.

By moving to the lower transit level, the conditional probabilities* of choosing the bus (B) or the rail (R), given the decision to travel by transit, become

$$P(B|T) = \frac{\exp(U_B)}{\exp(U_B)+\exp(U_R)} \qquad (1a)$$

$$P(R|T) = \frac{\exp(U_R)}{\exp(U_B)+\exp(U_R)} \qquad (1b)$$

To calculate the unconditional probabilities of choosing bus or rail, use the following equations:

$$P(B) = P(B|T)\ X\ P(T) \qquad (2a)$$

$$P(R) = P(R|T)\ X\ P(T) \qquad (2b)$$

Nested Logit Model cont.

The utility of the composite transit mode needs to capture the characteristics of all transit submodes (i.e., bus and rail). This is normally accomplished by including in the transit utility expression the Logsum variable (defined as the natural log of the denominator of Eq 1b.) multiplied by its calibration coefficient:

Logsum = ln {exp $(U_B) + \exp(U_R)\}$ (3)

The transit utility expression takes the form

$$U_T = a_T + \ \ldots + a_n \ \ X_n + \emptyset \ \ Logsum \quad (4)$$

The numerical value of the Logsum coefficient resulting from estimating the model provides information about the appropriateness of the selected nesting structure.

Implications of different values of Ø

- Ø = 0, (the two sub modes are perfect substitute of each other)
- Ø = 1 (there is an equivalent MNL, IIA property holds)
- Ø > 0 < 1 (acceptable nest)
- Ø > 1 (selected nesting structure is inappropriate)

An estimation procedure for a mode choice model of the nested logit structure shown in Fig.8.4.2 found that $U_T = a_T + \emptyset\ X\ Logsum\ with\ a_r = -0.52\ and\ \emptyset = 0.$ For a particular zonal interchange the following modal utilities were calculated in accordance with the estimated nested logit model:

$U_A = -0.26 \qquad U_B = -0.92 \qquad U_R = -0.82$

Calculate

(a) The corresponding mode shares

(b) The effect of a policy that is expected to a cause a change $\Delta U_B = -0.20.$

Solution

Part a:Baseline conditions

Nest level

Modem	U_m	exp(U_m)	P(m|T)
B	- 0.92	0.399	0.476
R	- 0.82	0.440	0.524
		$\sum =$ 0.839	1.000

$$U_T = -0.52 + 0\ X\ In(0.839) = -0.52$$

P (B) = P (B|T) X P (T) = 0.476 X 0.436 = 0.208
P (R) = P (R|T) X P (T) = 0.524 X 0.436 = 0.228
Whereas P(A) = 0.564
$\sum = 1.000$

Part b: After change
Nest level

Modem	U_m	exp(U_m)	P(m|T)
B	- 1.12	0.326	0.426
R	- 0.82	0.440	0.574
		$\sum$ = 0.766	1.000

$$U_T = -0.52 + 0\ X\ In\ (0.0.766) = -0.52$$

Primary choice level
Same as above:
P(A) = 0.564 and P(T) = 0.436

By Eqs. 8.4.14

P (B) = P (B|T) X P (T) = 0.476 X 0.426 = 0.186
P (R) = P (R|T) X P (T) = 0.524 X 0.574 = 0.250
Whereas P(A) = 0.564
$\sum = 1.000$

Ø = 1

Nest level

Mode m	U_m	$\exp(U_m)$	$P(m\|T)$
B	−0.88	0.415	0.475
R	−0.78	0.458	0.525
		$\Sigma = 0.873$	1.000

$$U_T = -0.52 + 1.0 \times \ln(0.839) = -0.56$$

Primary choice level

Mode m	U_m	$\exp(U_m)$	$P(m)$
A	−0.36	0.698	0.550
T	−0.56	0.571	0.450
		$\Sigma = 1.366$	1.000

By Eqs. 8.4.14

$$P(B) = P(B|T) \times P(T) = 0.475 \times 0.450 = 0.214$$

$$P(R) = P(R|T) \times P(T) = 0.525 \times 0.450 = 0.236$$

whereas

$$P(A) = 0.450$$

$$\Sigma = 1.000$$

Part b: After change

Nest level

Mode m	U_m	$\exp(U_m)$	$P(m\|T)$
B	−0.88	0.415	0.500
R	−0.88	0.415	0.500
		$\Sigma = 0.830$	1.000

$$U_T = -0.42 + 1.0 \times \ln(0.830) = -0.61$$

Primary choice level

Mode m	U_m	$\exp(U_m)$	$P(m)$
A	−0.36	0.698	0.462
T	−0.61	0.543	0.438
		$\Sigma = 1.241$	1.000

$$P(B) = P(B|T) \times P(T) = 0.438 \times 0.500 = 0.219$$

$$P(R) = P(R|T) \times P(T) = 0.438 \times 0.500 = 0.219$$

whereas

$$P(A) = \underline{0.562}$$

$$\Sigma = 1.000$$

Discussion The auto to bus share ratio before the change in the utility of the third mode (rail) was 0.550/0.214 = 2.57 and after the change 0.562/0.219 = 2.57. In other words, when $\theta = 1.0$, the IIA property holds true and an equivalent MNL model exists. According to Eq. 8.4.17, the corresponding MNL utilities become

$$U_A = -0.36 \qquad U_B = a_T - 0.88 = -1.30 \qquad U_R = a_T - 0.78 = -1.20$$

Applying the MNL to the baseline conditions using these modified utilities yields

Mode m	U_m	$\exp(U_m)$	$P(m)$
A	−0.36	0.698	0.550
B	−1.30	0.273	0.214
R	−1.20	0.301	0.236
		$\Sigma = 1.272$	1.000

This result is identical to that obtained via the equivalent nested model.

Ø =0.2

Part a: Baseline conditions

Nest level

Mode m	U_m	$\exp(U_m)$	$P(m\|T)$
B	-1.05	0.350	0.475
R	-0.95	0.387	0.525
		$\Sigma = 0.737$	1.000

$$U_T = -0.41 + 0.2 \times \ln(0.737) = -0.47$$

Primary choice level

Mode m	U_m	$\exp(U_m)$	$P(m)$
A	-0.41	0.644	0.515
T	-0.47	0.625	0.485
		$\Sigma = 1.289$	1.000

By Eqs. 8.4.14

$$P(B) = P(B|T) \times P(T) = 0.485 \times 0.475 = 0.230$$

$$P(R) = P(R|T) \times P(T) = 0.485 \times 0.425 = 0.255$$

whereas

$$P(A) = 0.515$$

$$\Sigma = 1.000$$

Part b: After change

Nest level

Mode m	U_m	$\exp(U_m)$	$P(m\|T)$
B	-1.35	0.259	0.401
R	-0.95	0.387	0.599
		$\Sigma = 0.646$	1.000

$$U_T = -0.41 + 0.2 \times \ln(0.646) = -0.50$$

Primary choice level

Mode m	U_m	$\exp(U_m)$	$P(m)$
A	-0.41	0.664	0.522
T	-0.61	0.608	0.478
		$\Sigma = 1.272$	1.000

By Eqs. 8.4.14

$$P(B) = P(B|T) \times P(T) = 0.478 \times 0.401 = 0.192$$

$$P(R) = P(R|T) \times P(T) = 0.478 \times 0.599 = 0.286$$

whereas

$$P(A) = 0.522$$

$$\Sigma = 1.000$$

Discussion In this case the before and after share ratios of auto and rail are, respectively, 0.515/0.255 = 0.202 and 0.522/0.286 = 1.83. The change in the utility of the third (bus) alternative did not affect the other two modes proportionately. With a value of θ between 0 and 1, the IIA property of the MNL model does not apply to the nested structure.

Optional Information

NL in practice

As a modelling tool the NL may be usefully presented in the following fashion (Ortúzar 1980b; Sobel 1980):

1. Its structure is characterised by grouping all subsets of correlated (or more similar) options in hierarchies or nests. Each nest, in turn, is represented by a *composite alternative* which competes with the others available to the individual (the example in Figure 7.2 considers two levels of nesting and four nests).
2. The introduction of information from lower nests in the next higher nests is done by means of the utilities of the composite alternatives; these are, by definition, equal to the expected maximum utility (EMU) of the options belonging to the nest and have the following expression:

$$\mathrm{EMU}_j = \log \sum_k \exp(V_k/\phi_j)$$

Therefore the composite utility of nest j is:

$$V_j = \phi_j \cdot \mathrm{EMU}_j$$

where ϕ_j are *structural* parameters to be estimated.

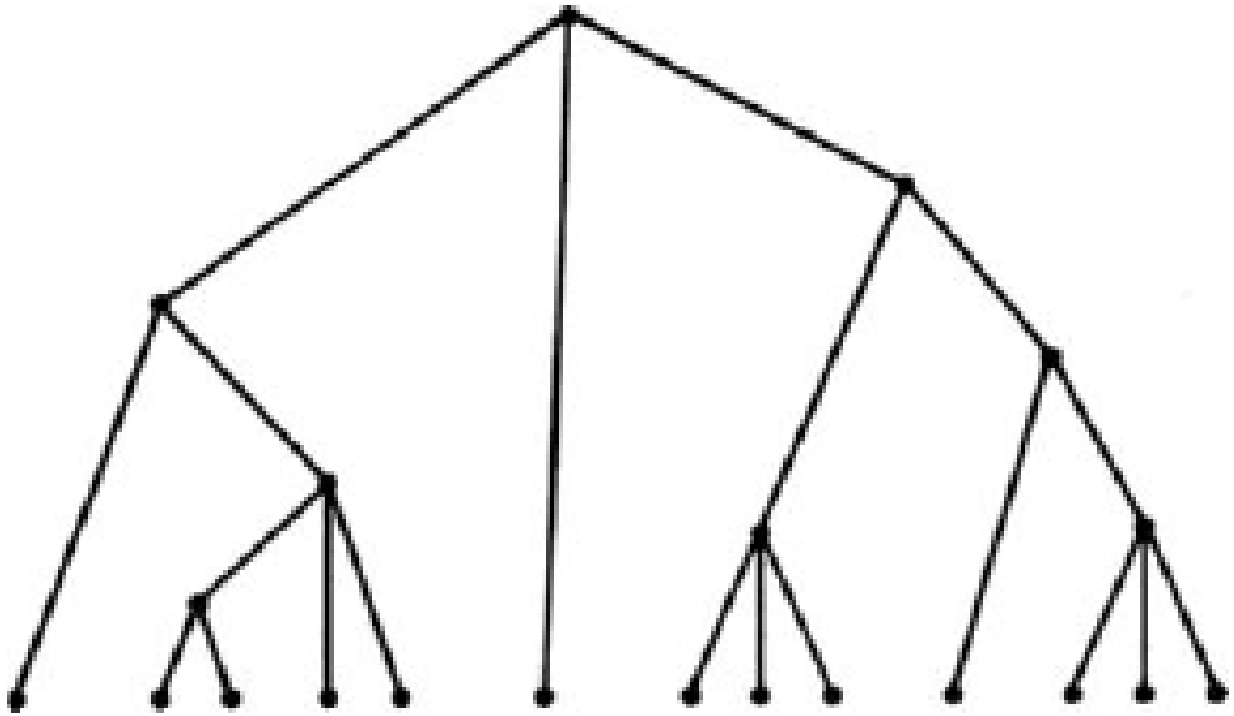
3. The probability that individual q selects option A_i in nest j may be computed as the product of the marginal probability of choosing the composite alternative N_J (in the higher nest) and the conditional probability of choosing option A_i in the lower nest, given that q selected the composite alternative.
4. If there is only one nest, the internal diagnosis condition (7.17) is expressed in this new notation as:

$$0 < \phi \leq 1 \qquad (7.24)$$

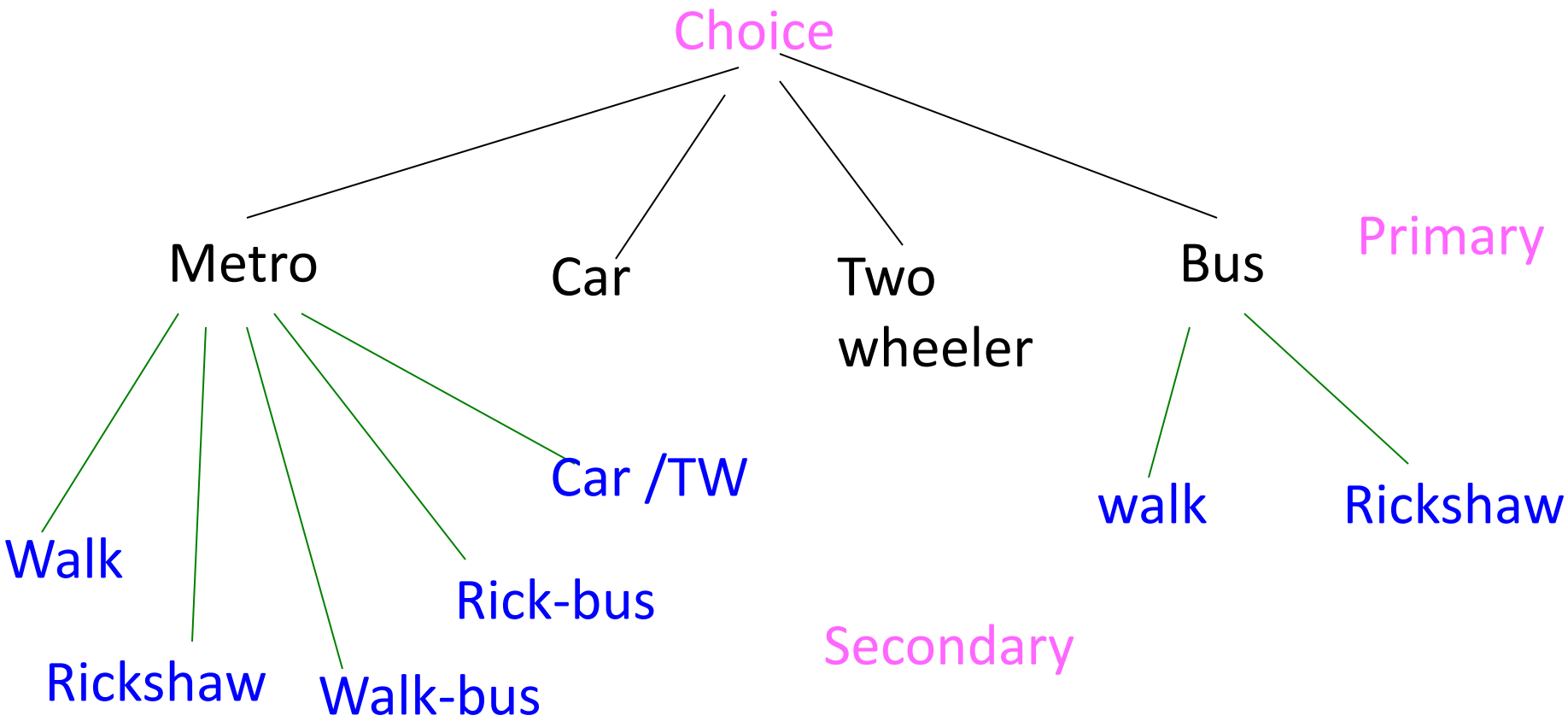
and let us briefly see why it needs to hold. If $\phi < 0$, an increase in the utility of an alternative in the nest, which should increase the value of EMU, would actually diminish the probability of selecting the nest; if $\phi = 0$, such an increase would not affect the nest's probability of being selected, as EMU would not affect the choice between car and public transport.

On the other hand, if $\phi > 1$ an increase in the utility of an alternative in the nest would tend to increase not only its selection probability but also those of the rest of the options in the nest (but note that the real reason is that β cannot be greater than λ as shown in expression 7.16). Finally, if $\phi = 1$ which is the equivalent to $\beta = \lambda$, the NL model becomes mathematically equivalent to the MNL; in such cases (i.e. when $\phi \approx 1$) it is more efficient to recalibrate the model as an MNL, as the latter has fewer parameters.

NL with several nests

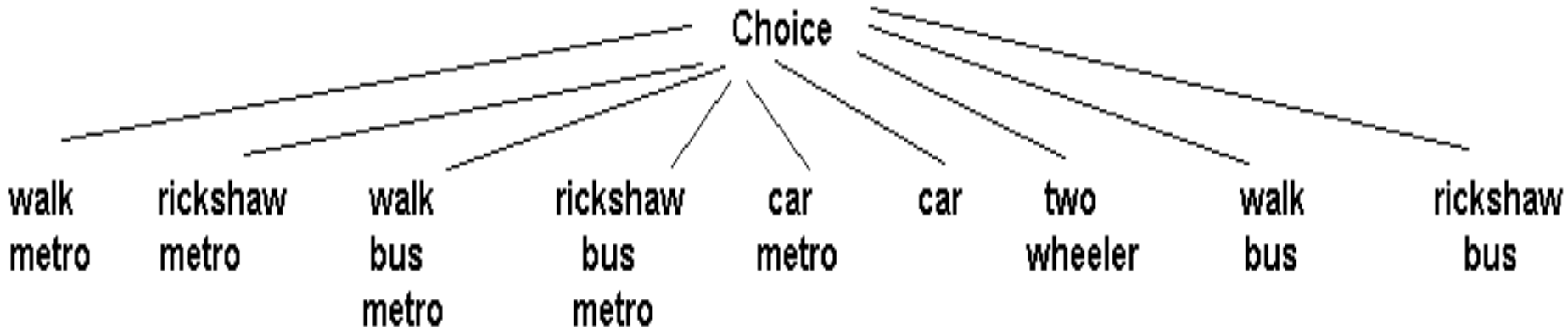


Mode options(Delhi Example, M. Advani Ph.D.Thesis)



Demand estimation model – mode choice options

1. Walk-metro
2. Rickshaw-metro (+walk)
3. Walk-bus-metro
4. Rickshaw-bus-metro (+walk)
5. Car or two wheeler-metro
6. Car
7. Two wheeler
8. Walk-bus
9. Rickshaw-bus (+walk)



- Utility equation for each mode is developed using Multinomial Logit model to calculate the probability of each mode being chosen.

- Significance and non-significance of different variables at 90% confidence level

·	Variable Description	Significance
1	AED (Access and egress trip distance)	Not significant
2	AET (Access and egress trip time)	significant
3	AEC (Access and egress trip cost)	significant
4	MHD (Main haul line trip distance)	Not significant
5	MHT (Main haul line trip time)	significant
6	MHC (Main haul line trip cost)	significant
7	TOTALD (Total trip distance)	Not significant
8	TOTALT (Total trip time)	Not significant
9	TOTALC (Total trip cost)	Not significant
10	AE_TD (ratio of access plus egress distance to total distance)	significant

·	Variable Description	Significance
11	AE_TT (ratio of access plus egress time to total time	Not significant
12	AE_TC (ratio of access plus egress cost to total cost	Not significant
13	AE_MHD (ratio of access plus egress distance to main haul line distance)	Not significant
14	AE_MHT (ratio of access plus egress time to main haul line time	significant

Adapting significant variables, developed utility function using maximum likelihood is,

Utility = β1 – 0.017AET - 0.024AEC -0.041MHT – 0.021MHC - 0.767AE_TD – 0.060AE_MHT

β1= Mode specific constant

AET = Time spent in making access and egress trips

AEC= Money spent in making access and egress trips (INR)

MHT= Time spent in main haul line trip

MHC= Money spent in making main haul line trip (INR)

AE_TD= Ratio of access, egress distance with total distance traveled

AE_MHT= Ration of time spent in access, egress trip with main haul line trip time

This utility equation has been used in calculating the probabilities attached in choosing different modes.

MNL to Nested – A need

IIA property is a major limitation of the MNL model as it implies equal competition between all pairs of alternatives.

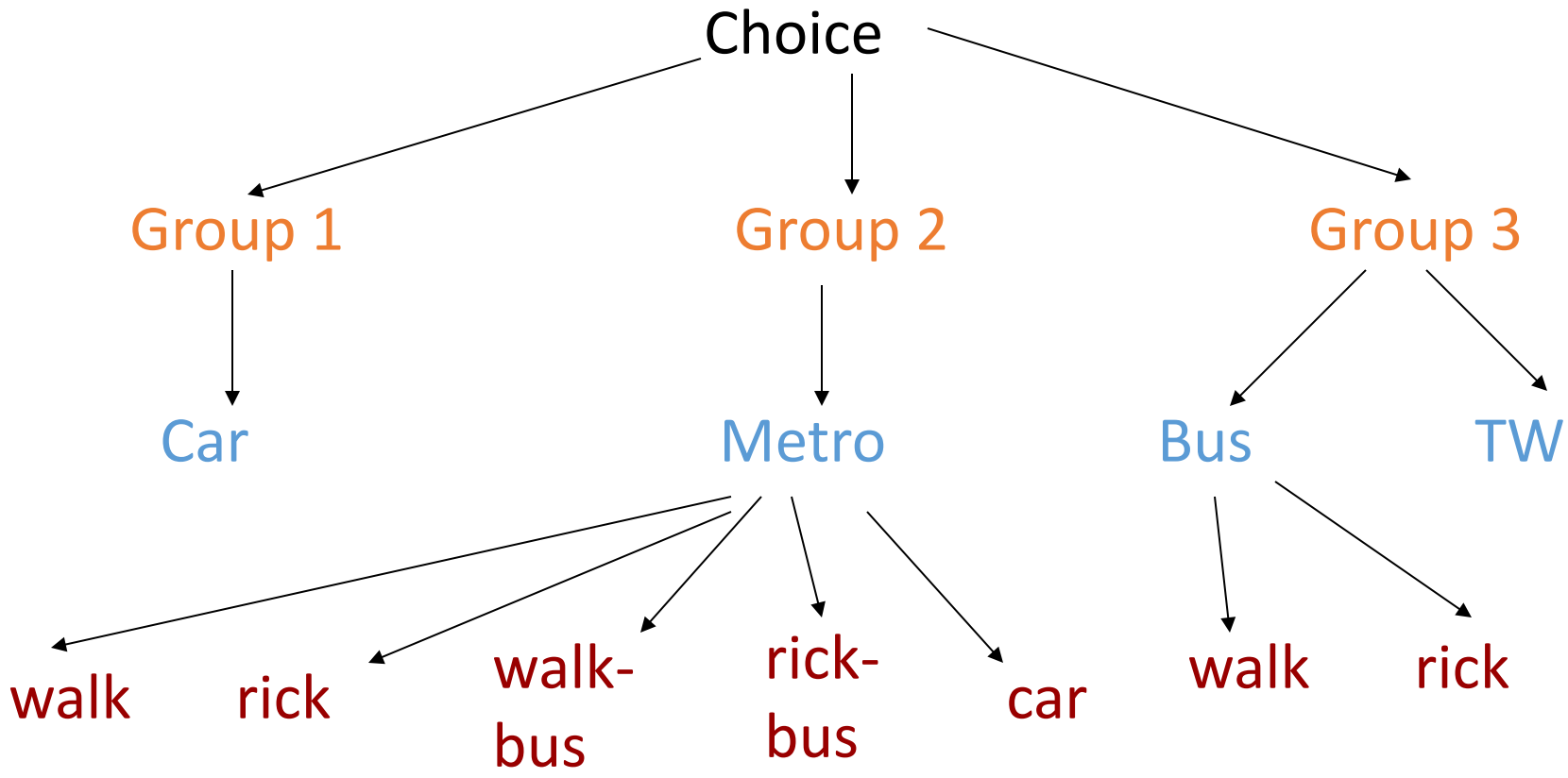
The NL model is characterized by grouping (or nesting) subsets of alternatives that are more similar to each other with respect to excluded characteristics than they are to other alternatives.

Sensitivity between different mode choices

Mode	Significance of Car ownership	Significance of TW ownership	Logsum of sub-modes (theta)	Decision
Metro & bus	significant	significant	Not accepted	Not accepted
Car & TW	significant	significant	Not accepted	Not accepted
Metro & car	significant	significant	Not accepted	Not accepted
Metro & TW	significant	significant	Not accepted	Not accepted

Sensitivity between different mode choices

Mode	Significance of car ownership	Significance of two wheeler ownership	Logsum of sub-modes (theta)	Acceptations decision
Bus &car	significant	not significant	not accepted	not accepted
Bus & TW	not significant	significant	Accepted	accepted
Metro, Car, bus & TW	significant	significant	Accepted	accepted
Metro, bus, TW & car	significant	significant	not accepted	not accepted



TW= Two wheeler, rick=Rickshaw

Calibration and Validation of the statistical model

- *Calibration*
  - 2001 modal shares (mode choice)
  - Metro phase1 ridership
  - *Trip length distribution (network)*

- *Validation*
  - On sample data testing (57 metro users, 57 bus users, 36 TW and 36 car users)
  - Chi-square test (McNemar)

Calibration

Modal share for the year 2001

Mode	RITES (2005) data	Model Calculations	Difference
Car	10.30	9.87	4.17%
TW	17.90	21.94	20.56%
Bus	59.80	56.19	6.03%
Other	12.00	12.00	0.00%

Discussion

- 35% trips would be feasible only if rickshaw is available to make their access and/or egress trips.

- Similarly, 45% trips would be dependent on bus for making feeder trips to metro.

Mode and destination choice

$$U(d,m) = U_d + U_{dm}$$

where, for example, U_d could correspond to that portion of utility specifically associated with the destination and U_{dm} to the disutility associated with the cost of travelling. If we write the expression above following our previous notation we get:

$$U(d,m) = V(d,m) + \varepsilon(d,m)$$

where

$$V(d,m) = V_d + V_{dm}$$

and

$$\varepsilon(d,m) = \varepsilon_d + \varepsilon_{dm}$$

It can be shown that if the residuals ε are separately IID, under certain conditions the Hierarchical or Nested Logit (NL) model (Williams 1977; Daly and Zachary 1978) is formed:

$$P(d,m) = \frac{\exp\{\beta(V_d + V_d^*)\}\exp(\lambda V_{dm})}{\sum_{d'}\exp\{\beta(V_{d'} + V_{d'}^*)\}\sum_{m'}\exp(\lambda V_{dm'})}$$

with

$$V_d^* = (1/\lambda)\log\sum_{m'}\exp(\lambda V_{dm'})$$

$\beta = \lambda$ (which occurs when $\varepsilon_d = 0$) the NL collapses, as special case, to the single parameter MNL. To understand why this is so, let us first write in full the utility expressions for the first destination in a simple binary mode case:

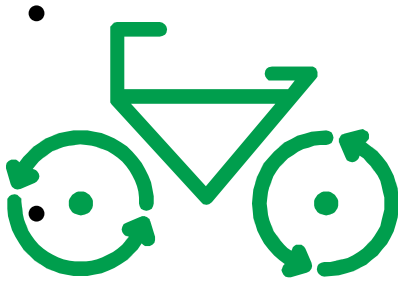
$$U(1,1) = V_1 + V_{11} + \varepsilon_1 + \varepsilon_{11}$$
$$U(1,2) = V_1 + V_{12} + \varepsilon_1 + \varepsilon_{12}$$

As can be seen, the source of correlation is the residual ε_1 which can be found in both U (1, 1) and U (1, 2); therefore when ε_d becomes 0, there is no correlation left and the model is indistinguishable from the MNL.

Travel mode comparisons

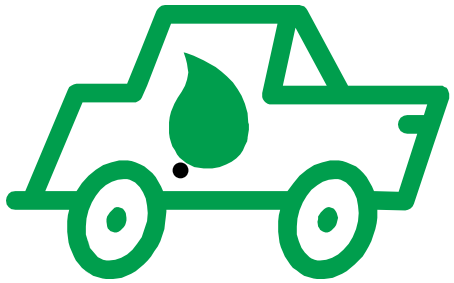
The societal concern

Energy Efficiency (megajoules/person trip)



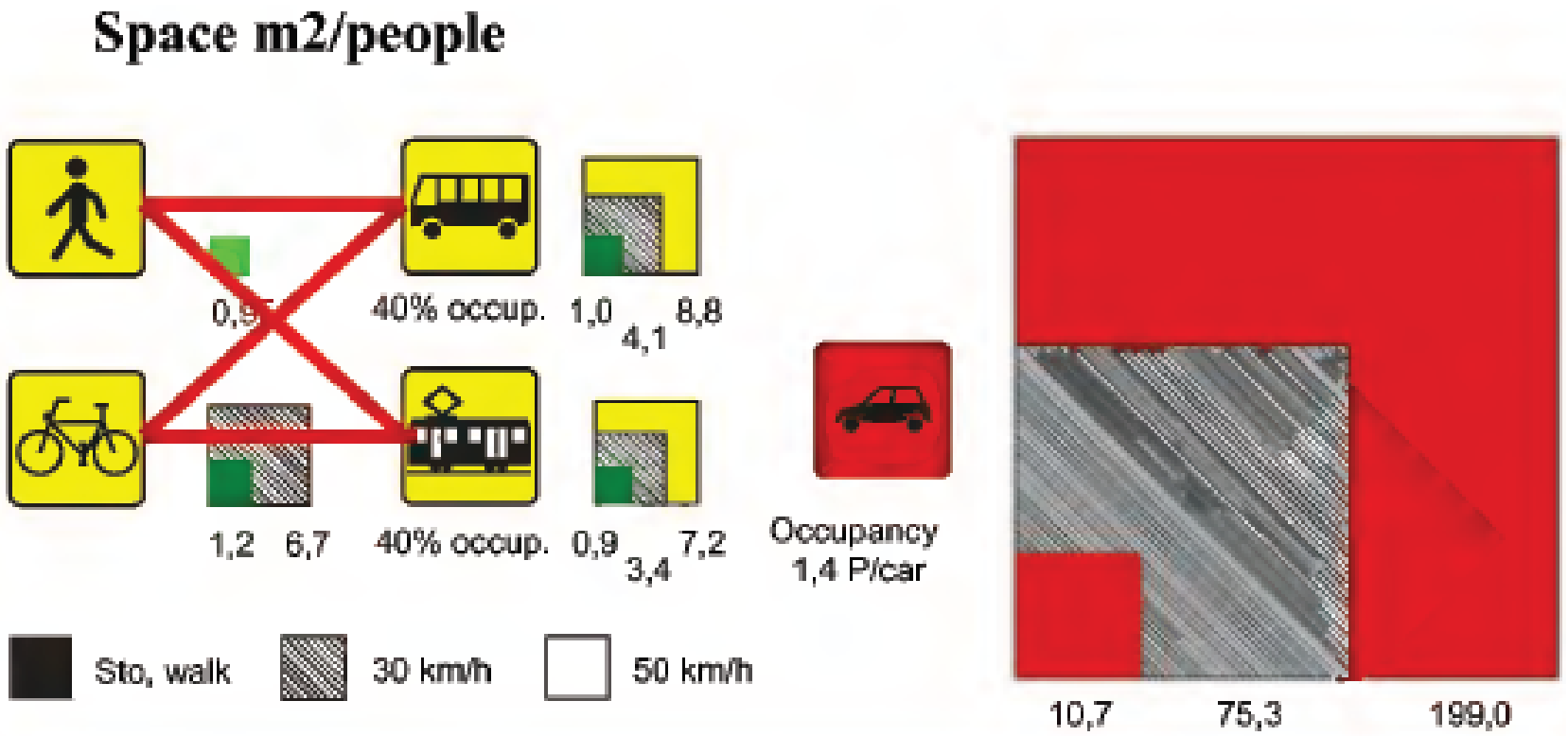
100 (0)

10 (5)



< 1 (40-60)

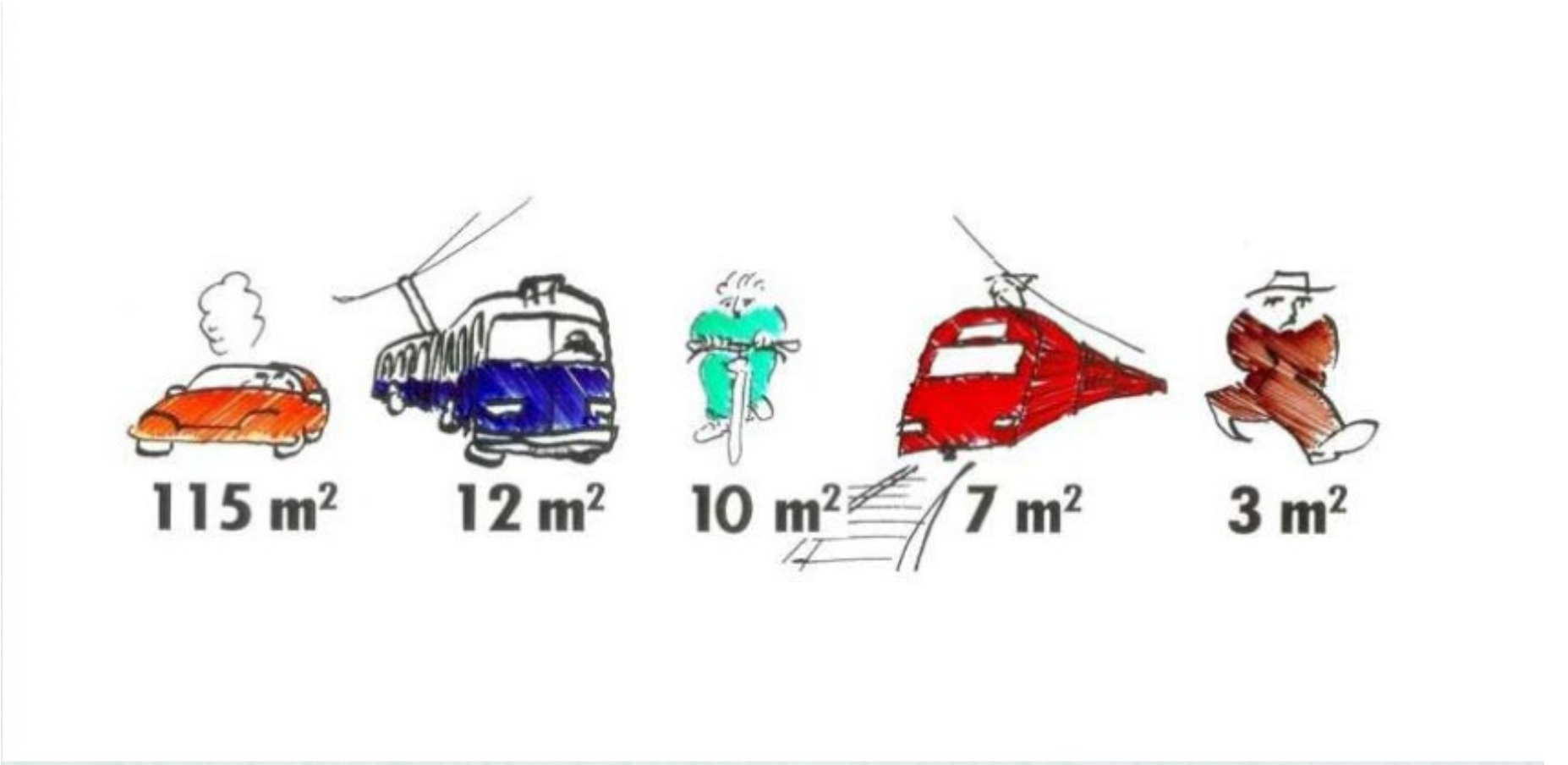
Efficient use of road space (H.Knoflacher)



Efficient
sustainable
Social connectivity high

inefficient use of space
not sustainable
No social connectivity

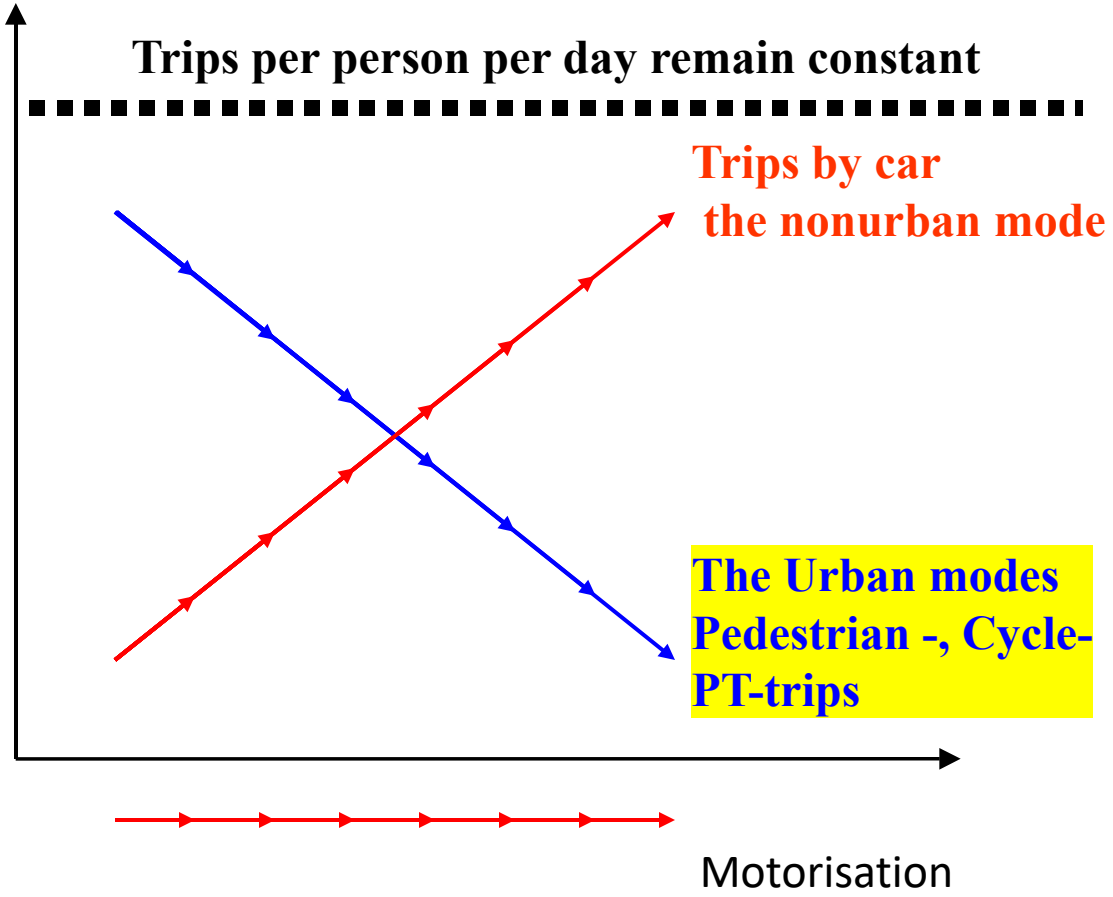
Amount of space used by different modes/person trip



Mobility does not change – only the modes (H.Knoflacher)

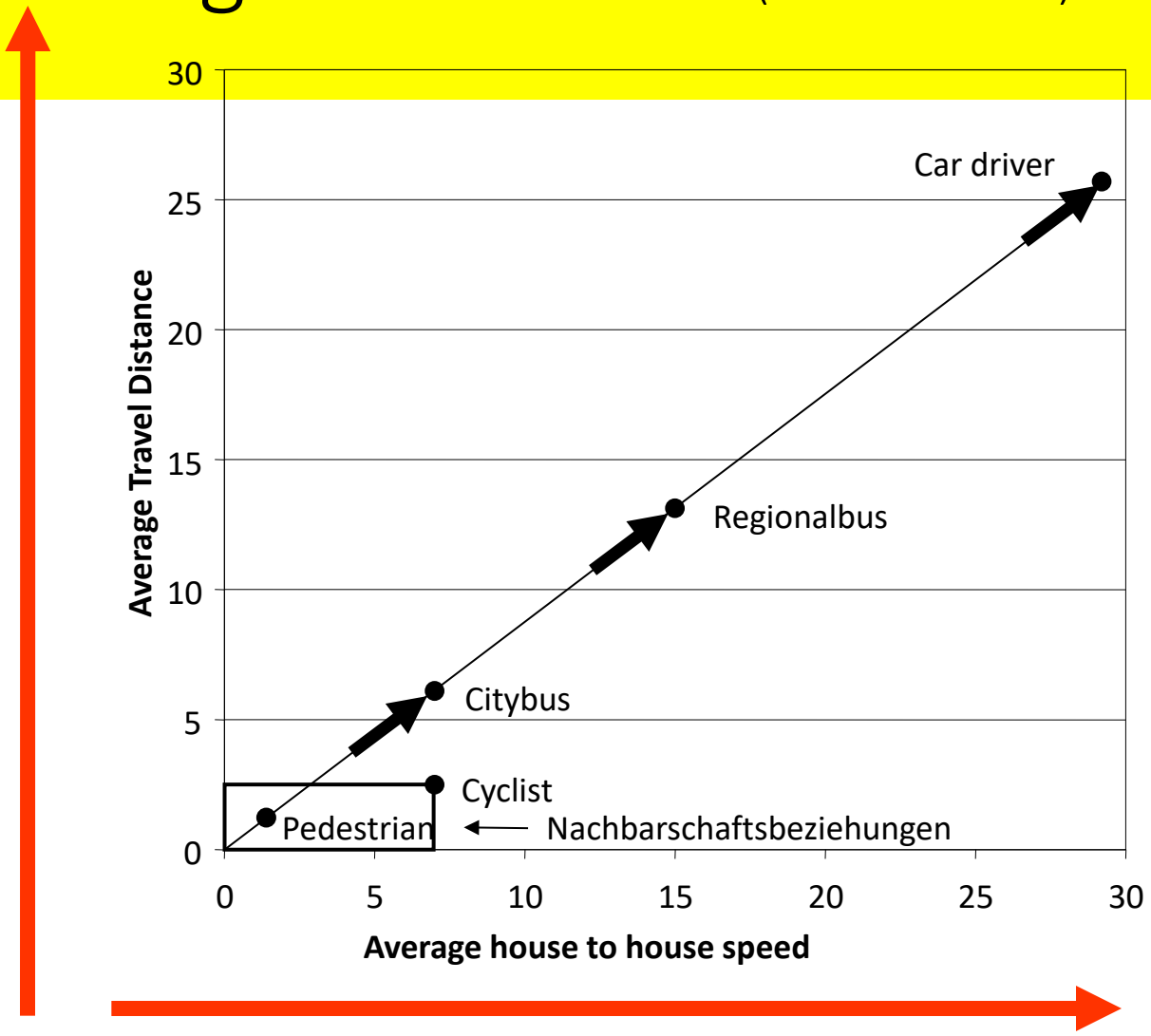
Trips per
Person per day

Reality

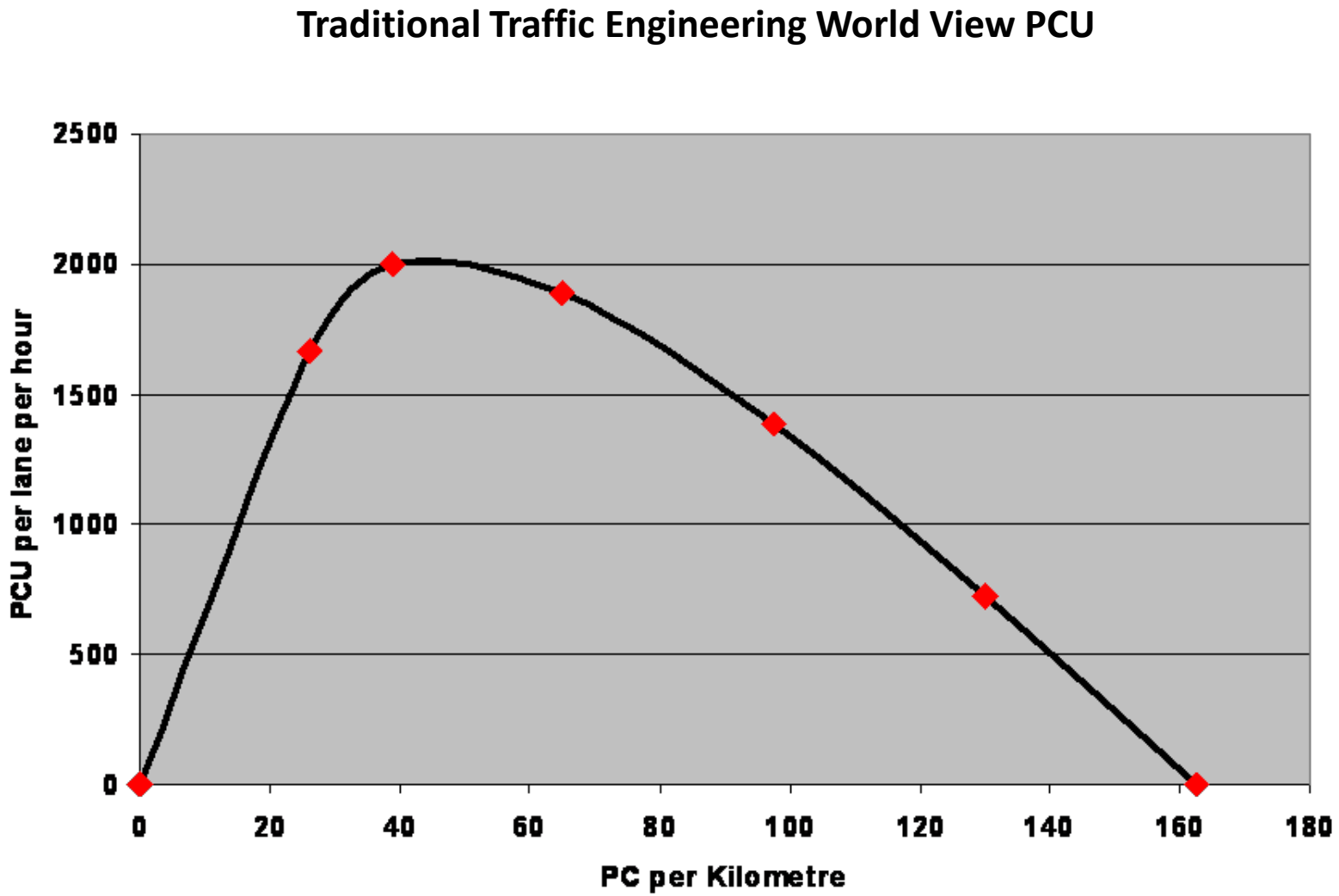


But why this change?

Higher speed – longer distances (H.Knoflacher)

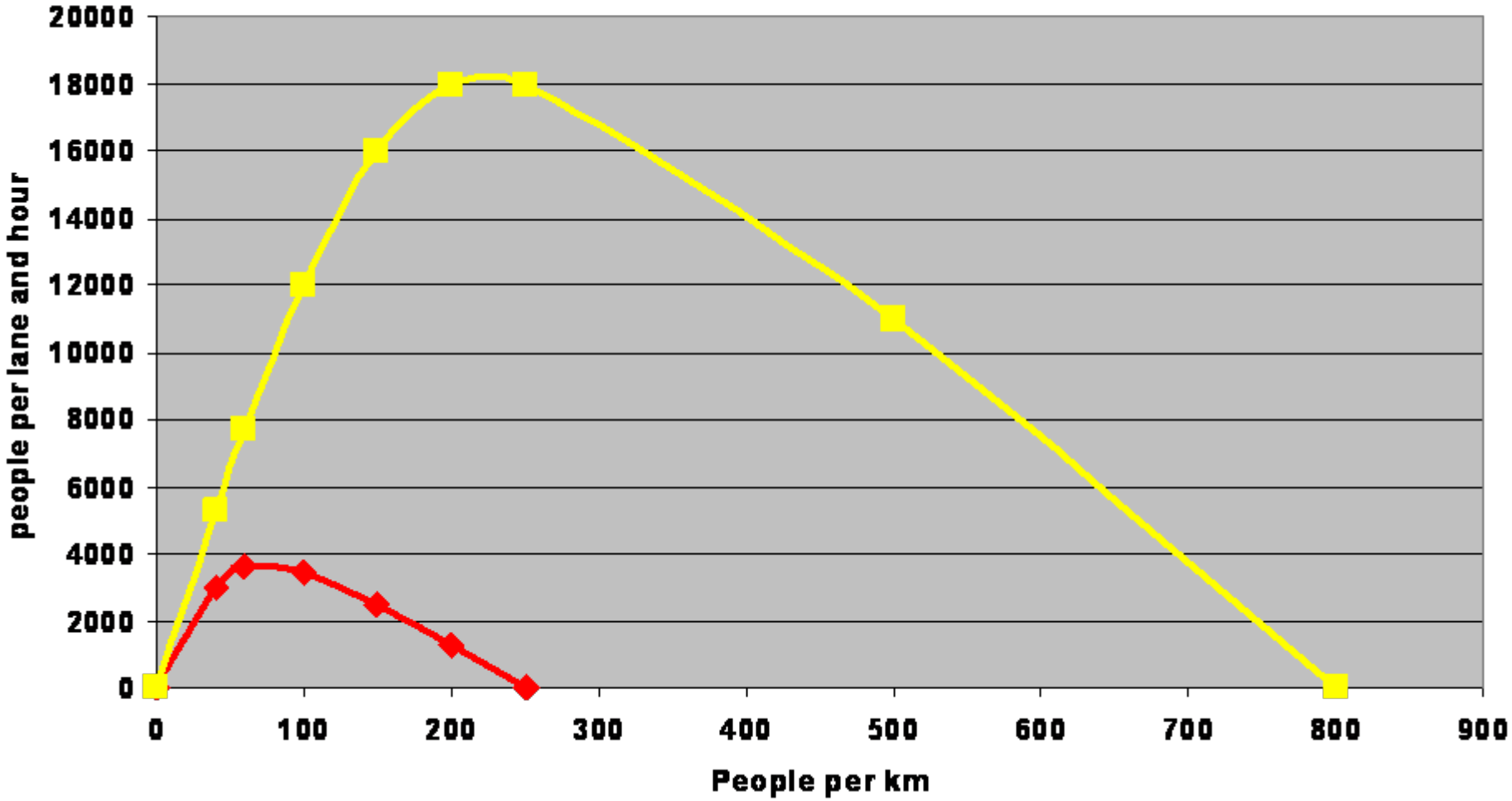


Best use of Public Space
1950 – 1970 the unit of the scale was the PCU?



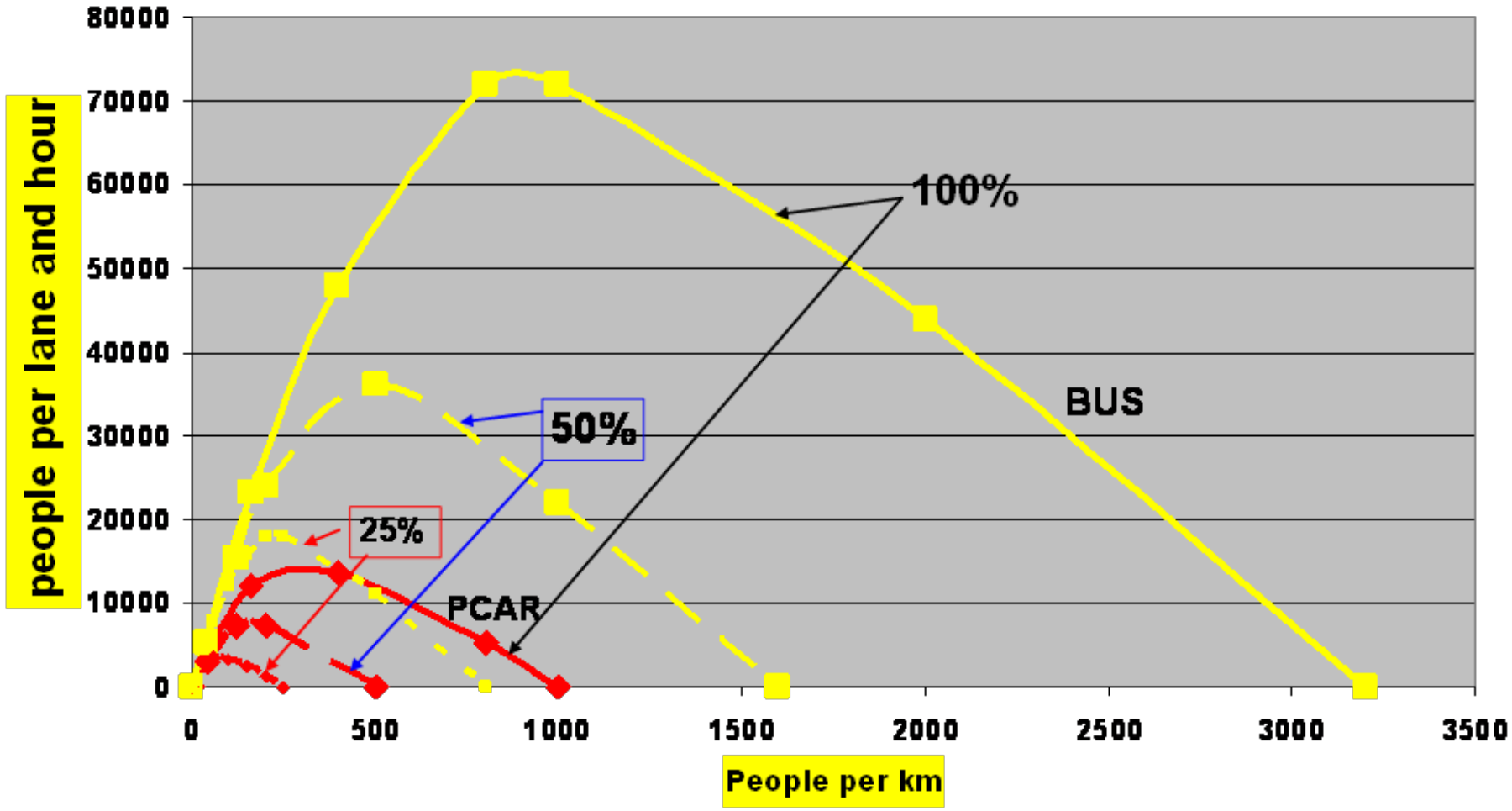
Euopean qualified Transport Planners after 1970 discovered
the roots of European Transport Planning – the people, the goods

Bus – PC 25% Occupancy



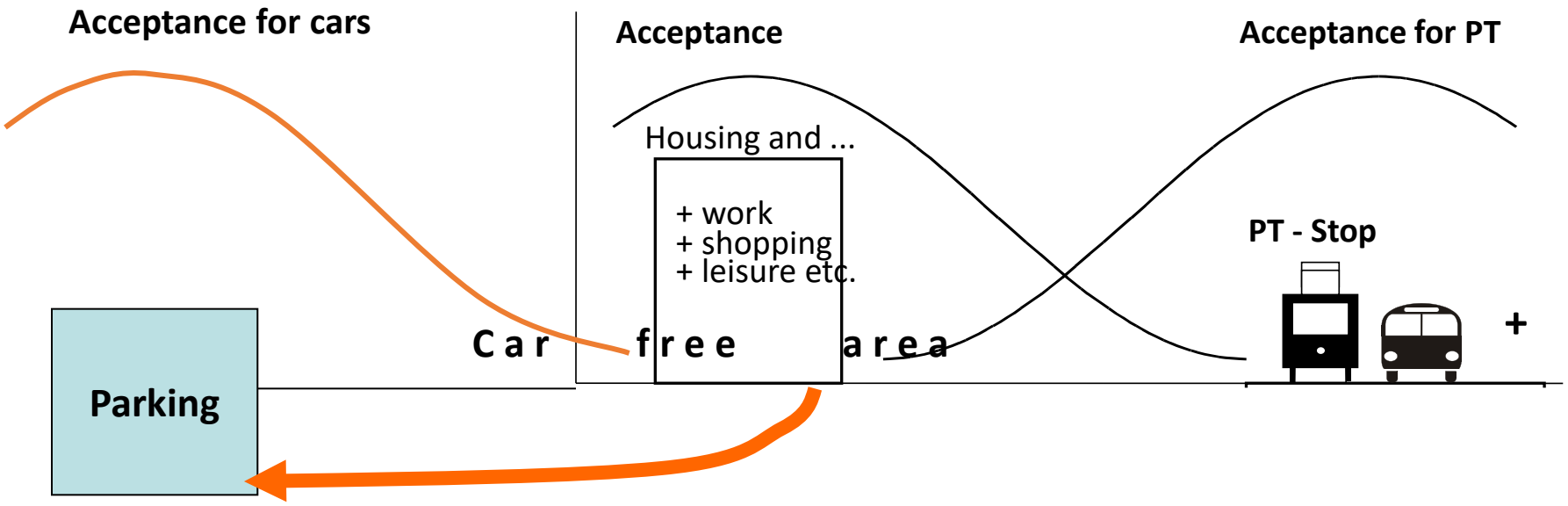
……and the real capacity of a lane

Bus – PC Occupancy rates 25%, 50%, 100%



Physical structure: Equity Car - PT

Precondition for a liveable city:



Equidistance to the Parking Place and the Public Transport Stop (PP = PTS)